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Bianchi Type IX Two Fluids Cosmological Models in General Relativity

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Abstract— In this paper we have studied Bianchi Type IX two fluids cosmological models with matter and radiating source. In the model one of the fluids represents the matter content of the universe and another fluid is the CMB radiation. To get the deterministic model, we have assumed a supplementary condition $a = b^n$ where a & b are metric potentials and *n* is the constant. We have also investigated the behaviors of some physical parameters.

Keywords: Bianchi Type-IX Space Time, CMB radiation, Two Fluids.

1 Introduction

THE cosmic microwave background (CMB) is one of the cornerstones of the homogeneous, isotropic model. Anisotropies in the CMB are related to small perturbation, superimposed on the perfectly smooth background, which are believed to seed formation of galaxies and large-scale structure in the universe. Coley and Dunn[1] (1990), have studied Bianchi type VI_0 model with two fluid source. Pant and Oli[2] (2002) has been investigated two fluid cosmological models using Bianchi type II space-time. Oli[3] (2008) presented anisotropic, homogeneous two fluid cosmological models in a Bianchi type I space time with a variable gravitational constant G and cosmological constant. Many researchers were investigated the Friedmann-Robertson-Walker models with two fluids source [Davidson[4] (1962), McIntosh[5] (1968), Coley and Tupper[6] (1986), Coley[7] (1988), Verma[8] (2009) etc.]

Recently Adhav[9] et al. (2011) examined two fluid cosmological models in Bianchi type V space-time. Here we have studied two-fluid models in Kantowski-Suchs space time. In these models we assume that both the fluids are present through out the evolution of the universe.

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2 Field Equations

We consider the metric in the form

$$ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)dy^2 + \left(b^2 \sin^2 y + a^2 \cos^2 y\right)dz^2 - 2a^2 \cos y dx dz \quad (1)$$

where *a* & *b* are functions of *t* alone.

The Einstein's field equation is

$$R^i_j - \frac{1}{2} g^i_j R = -T^i_j . \quad (2)$$

The energy momentum tensor for two fluids sources is given by

$$T_{ij} = T^{(m)}_{ij} + T^{(r)}_{ij} , \quad (3)$$

where $T^{(m)}_{ij}$ is the energy momentum tensor for matter field and $T^{(r)}_{ij}$ is the energy momentum tensor for radiation field which are given by

$$T^{(m)}_{ij} = (p_m + \rho_m) u^m_i u^m_j - p_m g_{ij} , \quad (4)$$

$$T^{(r)}_{ij} = \frac{4}{3}\rho_r u^r_i u^r_j - \frac{1}{3}\rho_r g_{ij} , \quad (5)$$

with $\quad g^{ij} u^m_i u^m_j = 1, \quad g^{ij} u^r_i u^r_j = 1. \quad (6)$

The off diagonal equations of (2) together with energy conditions imply that the matter and radiation are both comoving. We get,

$$u_i^{(m)} = (0,0,0,1), \qquad u_i^{(r)} = (0,0,0,1) \tag{7}$$

The Einstein's field equation (2) reduces to –

$$-\frac{3a^2}{4b^4} + \frac{2\ddot{b}}{b} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} = P_m + \frac{\rho_r}{3}, \tag{8}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + \frac{a^2}{4b^2} = P_m + \frac{\rho_r}{3}, \tag{9}$$

$$2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{a^2}{4b^4} = -\left(2P_m + \rho_m + \frac{2}{3}\rho_r\right), \tag{10}$$

$$P_m - \frac{\rho_r}{3} = 0. \tag{11}$$

3 SOLUTION OF THE FIELD EQUATION

Equations (8) to (11) are four independent equations in five unknowns a, b, ρ_r, ρ_m and p_m. For the complete determinacy of the system, we need extra conditions. We assume a relation in metric potentials as

$$a = b^n \tag{12}$$

when n is the constant.

Equating (9), (10) and (11) we get

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{3\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} = -2P_m - \rho_m \tag{13}$$

Equating (8), (9) and (11) we get

$$\frac{5\ddot{b}}{b} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} + \frac{3\ddot{a}}{a} + \frac{3\dot{a}\dot{b}}{ab} = 8P_m. \tag{14}$$

Equating (13) and (14) we get

$$\frac{\ddot{a}}{a} + \frac{2\ddot{b}}{b} = 5P_m + \frac{\rho_m}{2}. \tag{15}$$

Equating (8), (9) and (12) we get

$$2\ddot{b} + 2(1+n)\frac{\dot{b}^2}{b} = \frac{2b^{2n-3}}{(1-n)} - \frac{2}{b(1-n)} \tag{16}$$

We assume that

$$\dot{b} = f(b), \tag{17}$$

$$\ddot{b} = f\, f^1. \tag{18}$$

When

$$f^1 = \frac{df}{db}.$$

Equating (16), (17) and (18) we have

$$\frac{df^2}{db} + 2(1+n)\frac{f^2}{b} = \frac{2b^{2n-3}}{(1-n)} - \frac{2}{b(1-n)}. \tag{19}$$

Equation (19) leads to

$$f^2 = \frac{b^{2(n-1)}}{2n(1-n)} - \frac{1}{(1-n^2)} + \alpha b^{-2(1+n)}. \tag{20}$$

where α is integration constant.

Equating (14) and (15) we get

$$b^{2(n-1)} + \frac{2n}{(n+1)} + \frac{\alpha\, b^{-2(1+n)}}{2n(n-1)} = 10P_m + \rho_m. \tag{21}$$

The metric (1) reduces to the form

$$ds^2 = \left[\frac{T^{2(n-1)}}{2n(1-n)} - \frac{1}{(1-n^2)} + \alpha T^{-2(1+n)}\right]^{-1} dT^2 + T^{2n}dX^2 + T^2 dY^2 + \left(T^2\sin^2 Y + T^{2n}\cos^2 Y\right)dZ^2 - 2T^{2n}\cos Y dX dZ. \tag{22}$$

where $b = T$, $x = X$, $y = Y$ and $z = Z$

For the equation of state of matter we shall assume the γ-law

$$P_m = (\gamma - 1)\rho_m, \qquad 1 \le \gamma \le 2 \tag{23}$$

We get energy density matter, energy density of radiation and total energy density as

$$\rho_m = \frac{1}{(10\gamma - 9)}\left[T^{2(n-1)} + \frac{2n}{(n+1)} + \frac{\alpha T^{-2(1+n)}}{2n(n-1)}\right], \tag{24}$$

$$\rho_r = \frac{3(\gamma - 1)}{(10\gamma - 9)}\left[T^{2(n-1)} + \frac{2n}{(n+1)} + \frac{\alpha T^{-2(1+n)}}{2n(n-1)}\right], \tag{25}$$

$$\rho = \rho_r + \rho_m, \tag{26}$$

$$\rho = \frac{(3\gamma - 2)}{(10\gamma - 9)}\left[T^{2(n-1)} + \frac{2n}{(n+1)} + \frac{\alpha T^{-2(1+n)}}{2n(n-1)}\right]. \tag{27}$$

Case I: Dust model when $\gamma = 1$

The scalar of expansion, shear scalar and deceleration parameter are given by

$$\theta = 3H = (n+2)\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^{\frac{1}{2}}, \quad (28)$$

$$\sigma^2 = \frac{2}{3}(n^2+1)\left[\frac{T^{2n-4}}{2n(n-1)} - \frac{1}{T^2(1-n^2)} - \alpha T^{-2n-4}\right], \quad (29)$$

$$q = -1 - 6(n+2)^2(n+1)\left[\frac{T^{4n-8}}{4n(1-n^2)} - \frac{\alpha T^{-8}}{(1-n)} - \frac{T^{2n-6}}{n(1-n)(1-n^2)} - \alpha n T^{-2n-6} + \frac{1}{T^4(1-n^2)^2} - \alpha n T^{-4n-8}\right] \quad (30)$$

The density parameters are

$$\Omega_m = \frac{(n+2)^{-1}\left[T^{2n-1} + \frac{2n}{(n+1)} + \frac{\alpha T^{-2(1+n)}}{2n(n-1)}\right]}{3\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^2}, \quad (31)$$

$$\Omega_r = 0, \quad (32)$$

$$\Omega = \frac{(n+2)^{-1}\left[T^{2n-1} + \frac{2n}{(n+1)} + \frac{\alpha T^{-2(1+n)}}{2n(n-1)}\right]}{3\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^2}. \quad (33)$$

Case II: Radiation universe:

When $\gamma = \frac{4}{3}$

The scalar of expansion, shear scalar and deceleration parameter are given by

$$\theta = 3H = (n+2)\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^{\frac{1}{2}} \quad (34)$$

$$\sigma^2 = \frac{2(n^2+1)}{3}\left[\frac{T^{2n-4}}{2n(n-1)} - \frac{1}{T^2(1-n^2)} - \alpha T^{-2n-4}\right], \quad (35)$$

$$q = -1 - \sigma(n+2)^2(n+1)\left[\frac{T^{4n-8}}{4n(1-n^2)} - \frac{\alpha T^{-8}}{(1-n)} - \frac{T^{2n-6}}{(1-n)}\frac{T^{2n-6}}{n(1-n)(1-n^2)} - \alpha n T^{-2n-6} + \frac{1}{T^4(1-n^2)^2} - \alpha n T^{-4n-8}\right], \quad (36)$$

$$\Omega_m = \frac{1}{13(n+2)^2}\frac{\left[T^{2n-1} + \frac{2n}{(n+1)} + \frac{\alpha T^{-2(1+n)}}{2n(n-1)}\right]}{\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^2}, \quad (37)$$

$$\Omega_r = \frac{1}{13(n+2)^2}\frac{\left[T^{2n-1} + \frac{2n}{(n+1)} + \alpha . \frac{T^{-2(1+n)}}{2n(n-1)}\right]}{\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^2}, \quad (38)$$

$$\Omega = \frac{2}{13(n+2)^2}\frac{\left[T^{2n-1} + \frac{2n}{(n+1)} + \alpha . \frac{T^{-2(1+n)}}{2n(n-1)}\right]}{\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^2}. \quad (39)$$

Case III: Hard Universe: $\gamma \in \left(\frac{4}{3}, 2\right)$

Let $\gamma = \frac{5}{3}$

The scalar of expansion, shear scalar and deceleration parameter in hard universe are

$$\theta = 3H = (n+2)\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^{2(1-n^2)}} + \alpha T^{-2(n+2)}\right]^{\frac{1}{2}}, \quad (40)$$

$$\sigma^2 = \frac{2}{3}(n^2+1)\left[\frac{T^{2n-4}}{2n(n-1)} - \frac{1}{T^2(1-n^2)} - \alpha T^{-2n-4}\right], \quad (41)$$

$$q = -1 - 6(n+2)^2(n+1)\left[\frac{T^{4n-8}}{4n(1-n^2)} - \frac{\alpha T^{-8}}{(1-n)} - \frac{T^{2n-6}}{n(1-n)(1-n^2)} + \frac{1}{T^4(1-n^2)^2} - \alpha n T^{-4n-8}\right], \quad (42)$$

$$\Omega_m = \frac{1}{23(n+2)} \frac{\left[T^{2n-1} + \frac{2n}{(n+1)} + \frac{\alpha T^{-2(1+n)}}{2n(n-1)}\right]}{\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^2}, \quad (43)$$

$$\Omega_r = \frac{3}{23(n+2)} \frac{\left[T^{2n-1} + \frac{2n}{(n+1)} + \alpha.\frac{T^{-2(1+n)}}{2n(n-1)}\right]}{\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^2}, \quad (44)$$

$$\Omega = \frac{2}{23(n+2)} \frac{\left[T^{2n-1} + \frac{2n}{(n+1)} + \alpha.\frac{T^{-2(1+n)}}{2n(n-1)}\right]}{\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^2}. \quad (45)$$

Case IV: Zeldovich Universe:

When $\gamma = 2$

$$\theta = 3H = (n+2)\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^{\frac{1}{2}}, \quad (46)$$

$$\sigma^2 = \frac{2}{3}(n^2+1)\left[\frac{T^{2n-4}}{2n(n-1)} - \frac{1}{T^2(1-n^2)} - \alpha T^{-2n-4}\right], \quad (47)$$

$$q = -1 - 6(n+2)^2(n+1)\left[\frac{T^{4n-8}}{4n(1-n^2)} - \frac{\alpha T^{-8}}{(1-n)} - \frac{T^{2n-6}}{(1-n)} - \frac{T^{2n-6}}{n(1-n)(1-n^2)} - \alpha n T^{-2n-6}, + \frac{1}{T^4(1-n^2)^2} - \alpha n T^{-4n-8}\right] \quad (48)$$

$$\Omega_r = \Omega_m = \frac{1}{33(n+2)^2} \frac{\left[T^{2n-1} + \frac{2n}{(n+1)} + \frac{\alpha T^{-2(1+n)}}{2n(n-1)}\right]}{\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^2}, \quad (49)$$

$$\Omega = \Omega_m + \Omega_r,$$

$$\Omega = \frac{2}{33(n+2)^2} \frac{\left[T^{2n-1} + \frac{2n}{(n+1)} + \alpha.\frac{T^{-2(1+n)}}{2n(n-1)}\right]}{\left[\frac{T^{2n-4}}{2n(1-n)} - \frac{1}{T^2(1-n^2)} + \alpha T^{-2(n+2)}\right]^2}. \quad (50)$$

4 CONCLUSION

Here we observe that when $n = -2$ then expansion $\theta \to 0$ and $q = -1 < 0$ indicates the accelerating model of the universe. Also the energy density of matter $\Omega_m \to \infty$ and total energy $\Omega \to \infty$ but energy density of radiation $\Omega_r \to 0$ for dust model only.

In all cases $\lim_{T \to \infty} \left(\frac{\sigma}{\theta}\right)^2 = \frac{2(n^2+1)}{(n+1)^2} \neq 0$

Therefore these models do not approach isotropy for large value of T.

These models are expanding, the expansion in the model starts with big bang at $T = 0$.

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